# Year 12 Maths Methods Test 1, 2017

Differentiation Techniques and Applications of Differentiation

Name:____________________________________

**Section 1: Resource Free** ~~27~~ 30 **marks** ~~25~~ 30 **minutes**

QUESTION 1 [3, 1, 2, ~~4~~ 7 marks]

a) If $f(x) = \dfrac{1}{2x^2}$, evaluate $f''(-1)$

$f'(x) = -x^{-3}$ ✓

$f''(x) = 3x^{-4} = \dfrac{3}{x^4}$ ✓

$f''(-1) = 3$ ✓

b) Find g'(x), if $g(x) = (1+2x-2x^3)(x^2-1)$; do not simplify your answer

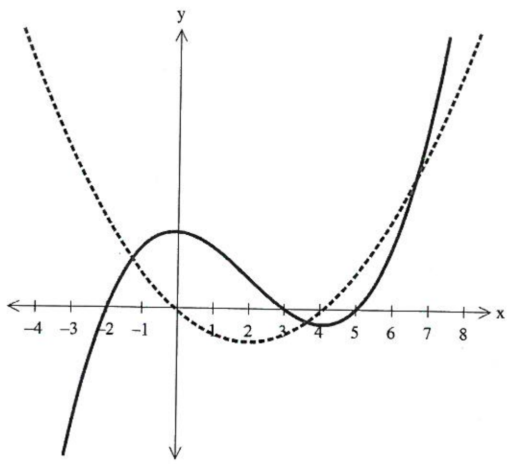
$(x^2-1)(2-6x^2)+(1+2x-2x^3)(2x)$ ✓

c) Use the chain rule to differentiate $\dfrac{2}{(x^3+2)^4}$ ; apply basic simplification

$-8(x^3+2)^{-5}(3x^2)$ ✓

$= \dfrac{-24x^2}{(x^3+2)^5}$ ✓

d) The 1[st] and 2[nd] derivative function of a function is shown. The x-coordinates of points where various features of the original function occur are shown below. State the nature of each of these points:




i. x = -2

minimum ✓ turning ✓ point

ii. x = 3

maximum ✓ turning ✓ point

iii. x = 4

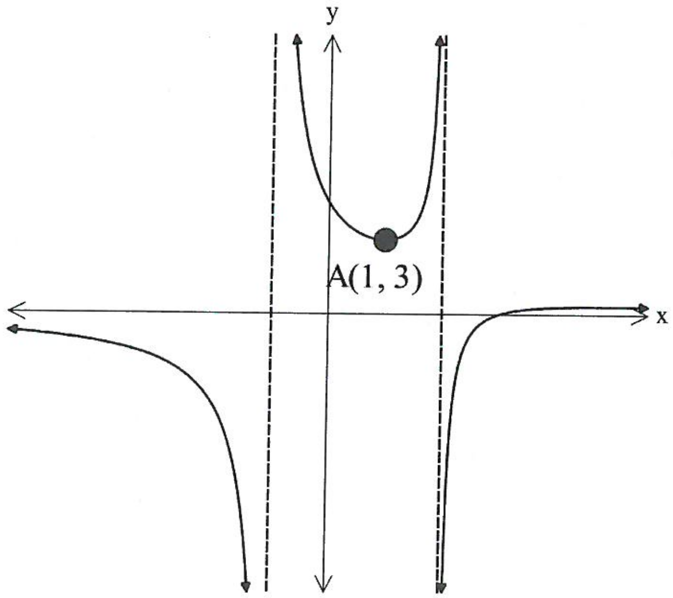
point of inflection ✓

iv. x = 5

minimum ✓ turning ✓ point

QUESTION 2 [3, 3 marks]

Consider the graph of $f(x) = \dfrac{3x-9}{x^2-x-2}$ shown below with a local minimum at A(1, 3)




a) Show that $f'(x) = \dfrac{-3(x-1)(x-5)}{(x^2-x-2)^2}$ $\qquad y=\frac{u}{v}$

$u = 3x-9 \qquad \frac{du}{dx} = 3$

$v = x^2-x-2 \qquad \frac{dv}{dx} = 2x-1$

$\frac{dy}{dx} = \dfrac{(x^2-x-2)3 - (3x-9)(2x-1)}{(x^2-x-2)^2}$ ✓✓

$= 3x^2-3x-6 - 6x^2+3x+18x-9 \,/\, (x^2-x-2)^2$

$= -3x^2+18x-15 \,/\, (x^2-x-2)^2$

$= -3(x^2-6x+5)/(x^2-x-2)^2$ ✓

$= \dfrac{-3(x-1)(x-5)}{(x^2-x-2)^2}$

b) Hence, or otherwise, determine the coordinates of the local maximum value of f(x).

$(x-1)(x-5)=0$ ✓ $\qquad \dfrac{3(5)-9}{(5)^2-5-2} = \dfrac{6}{18} = \dfrac{1}{3}$

$x=1 \qquad x=5$ ✓

$x=1$ is local min

$(5, \frac{1}{3})$ is local max ✓

QUESTION 3 [3 marks]

The volume of a solid sphere is given by $\frac{4}{3}\pi r^3$ where r is the radius. If the radius is increased from 2 cm to 2.1 cm, use the incremental formula to find the approximate increase in volume. Give your answer simplified in terms of $\pi$.

$\frac{dV}{dr} = 4\pi r^2$ ✓

$\delta V \approx 4\pi r^2 \delta r$ ✓

$\delta V \approx 4\pi 4 (0.1)$ ✓

$\delta V \approx 1.6\pi \text{ cm}^3$

$\frac{8\pi}{5}$

QUESTION 4 [1, 2, 3, 2 marks]

The motion of a body is determined by $x = t^3 - 3t^2 - 9t + 4$, where x is measured in cm and t is in seconds. Find

a) The velocity-time equation

$V = 3t^2 - 6t - 9$ ✓

b) The acceleration-time equation

$a = 6t - 6$ ✓✓

c) The time when the body is at rest

$0 = 3(t^2 - 2t - 3)$

$0 = 3(t-3)(t+1)$

$t = 3, -1$

$t = 3$ seconds

$v = 0$ ✓

sol'ns ✓

$t = 3$ sec ✓

d) The acceleration when the body is at rest

$a = 6(3) - 6$ ✓

$a = 12 \text{ cm/s}^2$ ✓

# Year 12 Maths Methods Test 1, 2017

## Differentiation Techniques and Applications of Differentiation

CANNING VALE COLLEGE

Name:______________________________

**Section 2: Resource Rich** **~~30~~ 25 marks** **~~30~~ 25 minutes**

QUESTION 5 [4 marks]

The graph of the function with rule $y = \frac{k}{2(x^3+1)}$ has gradient 1 when $x = 1$. Find the value of $k$.

$\frac{dy}{dx} = -\frac{1}{2}k(x^3+1)^{-2}.(3x^2)$

$= \frac{-3x^2k}{2(x^3+1)^2}$

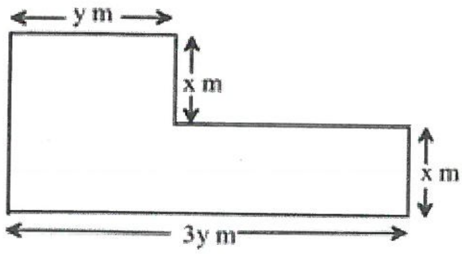
$1 = -\frac{3(1)^2k}{2(1^3+1)^2}$

$1 = -\frac{3k}{8}$

$k = -\frac{8}{3}$

QUESTION 6 [1, 1, 1, 2 marks]

A flower bed is to be L-shaped, as shown in the diagram. Its perimeter is 48 m




a) Write down an expression for the area, A $m^2$, in terms of $y$ and $x$

$A = 3xy + xy$

$= 4xy$

b) Find $y$ in terms of $x$

$6y + 4x = 48$

$y = \frac{(48-4x)}{6}$

c) Write down an expression for A in terms of $x$.

$A = \frac{4x(48-4x)}{6}$

d) Find the values of $x$ and $y$ that give the maximum area

$\frac{dA}{dx} = \frac{\cancel{-16(x} - (16x-96)}{3}$

$\frac{-(16x-96)}{3} = 0$

$x = 6$

$x = 6m \quad y = 4$

QUESTION 7 [3 marks]

A coat of paint of thickness 0.05 cm is to be applied uniformly to the faces of a cube of edge 30 cm. Use calculus methods to find the amount of paint required for the job.

$V = l^3$

$\frac{dV}{dl} = 3l^2$

$\delta V = 3l^2 \delta l$

$\delta V = 3(30)^2(0.1)$

$= 270 cm^3$

$\therefore$ need 270mL of paint.

QUESTION 8 [4 marks]

The length of time, in seconds, a certain individual takes to learn a list of n items is approximated by $f(x) = 4n\sqrt{n-4}$ . Use calculus to find the percentage increase in time taken when the number of items in the list is increased ~~by 1%~~ from 85 to 90 items.

$y = 4n\sqrt{n-4}$

$\cancel{\frac{dy}{dx} = 6n - 16}$

$\frac{dy}{dn} = (6n-16)(n-4)^{-1/2}$

$\cancel{\frac{\delta y}{y} \approx \frac{(6n-16)(n-4)^{-1/2} \cdot \delta n}{4n(n-4)^{1/2}}}$

$\cancel{\approx \frac{6n-16 \; \delta n}{4n(n-4)}}$

$\delta y = (6n-16)(n-4)^{-1/2} \cdot 5$

$= (6(85-16))(85-16)^{-1/2}(5)$

$= \cancel{3.6}\text{ sec. } 274 \text{ sec.}$

QUESTION 9 [2, 1, 4 marks]

A POLYNOMIAL FUNCTION $f(x) = ax^4 + bx^2 + c$, where a, b and c are real constants, has the following features:

- f(x) = 0 only for x = -2 and x = 2
- f'(x) = 0 only for x = -1, x = 0 and x = 1
- f'(x) > 0 only for -1 < x < 0 and x > 1
- f''(0) < 0

a) At the point where the curve intersects the y-axis, is it concave up or concave down? Explain your answer

Concave down ✓

$f''(0) < 0$ ✓

b) Is c positive or negative? Explain your answer

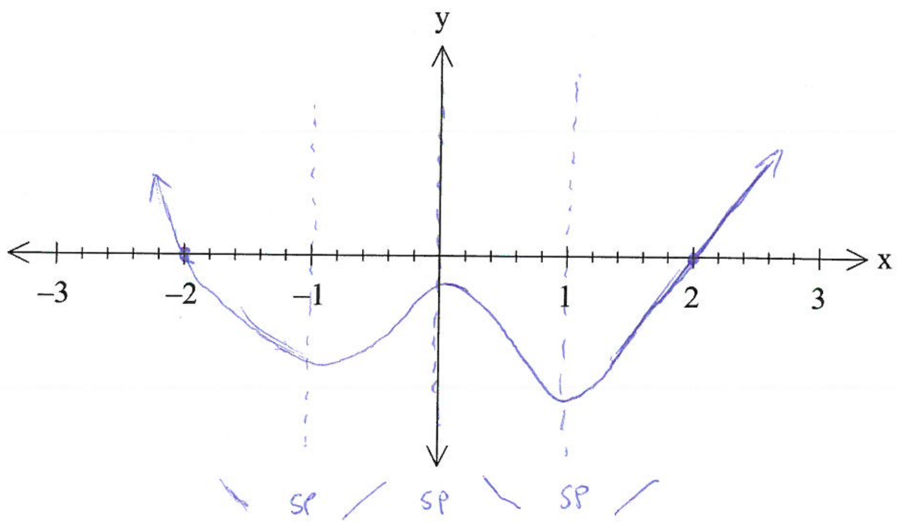
negative ✓

From -1 to 0, positive gradient

after 0, negative gradient

Doesn't cross y-axis

c) Sketch a possible graph of the function on the axes below

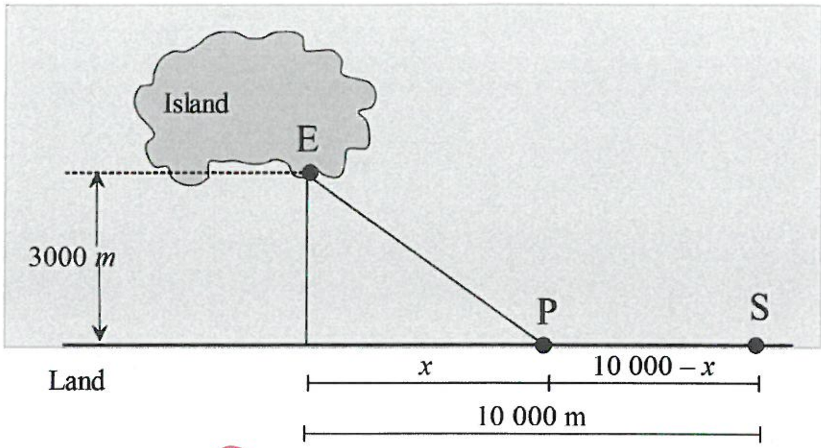



✓ - roots

✓ - turning points

✓✓ - gradients

Question 10 [1, 2, 3 marks]




In the accompanying diagram, S represents the position of a power relay station located on a straight coast and E shows the location of a marine biology experimental station on an island. A cable is to be laid connecting the relay station with the experimental station. The cost of running cable on land is $1.50 per metre and the cost of running the cable under water is I $2.50 per metre. Locate the point P that will result in a minimum cost.

a) State the distance from E to P in terms of $x$

$\sqrt{3000^2 + x^2}$ ✓

b) State the cost of the cabling in terms of $x$

$Cost = 1.5(10\,000 - x) + 2.5\left(\sqrt{3000^2 + x^2}\right)$ ✓ ✓

c) Find the value of $x$ that will minimise the cost

$$\frac{dC}{dx} = \frac{5x - 3\sqrt{3000^2 + x^2}}{2\sqrt{3000^2 + x^2}} = 0$$

✓ - derivative

✓ - = 0

$x = 2250$

✓ - solution